Fuzzy logic
What Is Fuzzy Logic?

- Form of multi-valued logic (algebra) derived from fuzzy set theory.
- Designed to deal with reasoning that is approximate rather than accurate.
- Consequence of the 1965 proposal of fuzzy set theory by Lotfi Zadeh.
- In contrast with "crisp logic", where binary sets have binary logic, fuzzy logic variables may have a truth value that ranges between 0 and 1.
- Can include linguistic variables, like: high, low, hot, cold, and very.
Fuzzy logic

- Fuzzy logic addresses key problem in expert systems
  - How to represent domain knowledge
  - Humans use imprecisely calibrated terms
  - How to build decision trees on imprecise thresholds
Examples where fuzzy logic is used

- Automobile and other vehicle subsystems, such as ABS and cruise control (e.g. Tokyo monorail)
- Air conditioners
- The Massive engine used in the *Lord of the Rings* films, which helped show huge scale armies create random, yet orderly movements
- Cameras
- Digital image processing, such as edge detection
- Rice cookers
- Dishwashers
- Elevators
- Washing machines and other home appliances
- Video game artificial intelligence
- Massage boards and chat rooms
- Fuzzy logic has also been incorporated into some microcontrollers and microprocessors, for instance Freescale 68HC12.
In the city of Sendai in Japan, a 16-station subway system is controlled by a fuzzy computer (Seiji Yasunobu and Soji Miyamoto of Hitachi) – the ride is so smooth, riders do not need to hold straps.

- **Nissan** – fuzzy automatic transmission, fuzzy anti-skid braking system
- **CSK, Hitachi** – Hand-writing Recognition
- **Sony** - Hand-printed character recognition
- **Ricoh, Hitachi** – Voice recognition
- Tokyo’s stock market has had at least one stock-trading portfolio based on Fuzzy Logic that outperformed the Nikkei exchange average.
Intel Corporation's Embedded Microcomputer Division Fuzzy Logic Operation

- http://www.intel.com/design/mcs96/designex/2351.htm

Motorola 68HC12 MCU
For washing machines, Fuzzy Logic control is almost becoming a standard feature. Fuzzy controllers to load-weight, fabric-mix, and dirt sensors and automatically set the wash cycle for the best use of power, water, and detergent.

- **GE WPRB9110WH Top Load Washer**
- Others: Samsung, Toshiba, National, Matsushita, etc.
- **Haier ESL-T21 Top Load Washer**
- **Miele WT945 Front Load All-in-One Washer / Dryer**
- **AEG LL1610 Front Load Washer**
- **Zanussi ZWF1430W Front Load Washer**
Worth to read...

• http://www.fuzzytech.com/e/e_a_esa.html

• http://www.fuzzytech.com/e/e_a_plc.html
• In 1965, Lotfi A. Zadeh of the University of California at Berkeley published "Fuzzy Sets," which laid out the mathematics of fuzzy set theory and, by extension, fuzzy logic.

• Zadeh had observed that conventional computer logic couldn't manipulate data that represented subjective or vague ideas, so he created fuzzy logic to allow computers to determine the distinctions among data with shades of gray, similar to the process of human reasoning.
What is fuzzy logic?

• Definition of fuzzy
  • Fuzzy – “not clear, distinct, or precise; blurred”

• Definition of fuzzy logic
  • A form of knowledge representation suitable for notions that cannot be defined precisely, but which depend upon their contexts.
Why use fuzzy logic?

Pros:
• Conceptually easy to understand w/ “natural” maths
• Tolerant of imprecise data
• Universal approximation: can model arbitrary nonlinear functions
• Intuitive
• Based on linguistic terms
• Convenient way to express expert and common sense knowledge

Cons:
• Not a cure-all
• Crisp/precise models can be more efficient and even convenient
• Other approaches might be formally verified to work
Fuzzy sets and concepts are commonly used in natural language
John is **tall**
Dan is **smart**
Alex is **happy**
The class is **hot**
• E.g., the crisp set **Tall** can be defined as:

\[ \{ x \mid \text{height } x > 1.8 \text{ meters} \} \]

But what about a person with a height = 1.79 meters?
What about 1.78 meters?
...
What about 1.52 meters?
In a fuzzy set a person with a height of 1.8 meters would be considered tall to a **high degree**
A person with a height of 1.7 meters would be considered tall to a lesser degree etc.
- The function can change for basketball players, Danes, women, children etc.
In traditional set theory, an element either belongs to a set, or it does not.

In Fuzzy Set Theory membership functions classify elements in the range \([0,1]\), with 0 and 1 being no and full inclusion, the other values being partial membership.
Simple example of Fuzzy Logic

Controlling a fan:

Conventional model –
if temperature > X, run fan
else, stop fan

Fuzzy System -
if temperature = hot, run fan at full speed
if temperature = warm, run fan at moderate speed
if temperature = comfortable, maintain fan speed
if temperature = cool, slow fan
if temperature = cold, stop fan
bool speed;
get the speed
if ( speed == 0) {
    // speed is slow
}
else {
    // speed is fast
}
Better (Fuzzy Representation)

- For every problem must represent in terms of fuzzy sets.

- What are fuzzy sets?

Slowest
[ 0.0 – 0.25 ]

Slow
[ 0.25 – 0.50 ]

Fast
[ 0.50 – 0.75 ]

Fastest
[ 0.75 – 1.00 ]
Representing Fuzzy Sets

float speed;
get the speed
if ((speed >= 0.0)&&(speed < 0.25)) {
  // speed is slowest
}
else if ((speed >= 0.25)&&(speed < 0.5)) {
  // speed is slow
}
else if ((speed >= 0.5)&&(speed < 0.75)) {
  // speed is fast
}
else // speed >= 0.75 && speed < 1.0 {
  // speed is fastest
}
Range of logical values in Boolean and fuzzy logic

(a) Boolean Logic.

(b) Multi-valued Logic.
The classical example in fuzzy sets is *tall men*. The elements of the fuzzy set “*tall men*” are all men, but their degrees of membership depend on their height.

<table>
<thead>
<tr>
<th>Name</th>
<th>Height, cm</th>
<th>Degree of Membership</th>
<th>Crisp</th>
<th>Fuzzy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chris</td>
<td>208</td>
<td>1</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>Mark</td>
<td>205</td>
<td>1</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>John</td>
<td>198</td>
<td>1</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td>Tom</td>
<td>181</td>
<td>1</td>
<td>0.82</td>
<td></td>
</tr>
<tr>
<td>David</td>
<td>179</td>
<td>0</td>
<td>0.78</td>
<td></td>
</tr>
<tr>
<td>Mike</td>
<td>172</td>
<td>0</td>
<td>0.24</td>
<td></td>
</tr>
<tr>
<td>Bob</td>
<td>167</td>
<td>0</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>Steven</td>
<td>158</td>
<td>0</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>Bill</td>
<td>155</td>
<td>0</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>Peter</td>
<td>152</td>
<td>0</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>
Crisp and fuzzy sets of “tall men”
How to represent a fuzzy set in a computer?

- First, we determine the membership functions. In our “tall men” example, we can obtain fuzzy sets of tall, short and average men.
- The universe of discourse – the men’s heights – consists of three sets: short, average and tall men. As you will see, a man who is 184 cm tall is a member of the average men set with a degree of membership of 0.1, and at the same time, he is also a member of the tall men set with a degree of 0.4.
Crisp and fuzzy sets of short, average and tall men

Crisp Sets

Degree of Membership

Height, cm

Fuzzy Sets

Degree of Membership

Height, cm
Membership Functions for $T(temperature) = \{too-cold, cold, warm, hot, too-hot\}$.
Different Types of Membership Functions.

There are different forms of membership functions such as triangular, trapezoidal, Gaussian, or singleton. The most common types of membership functions are triangular, trapezoidal, and Gaussian shapes.
\[
\mu_A(x) = \begin{cases} 
0 & x < a \\
\frac{x-a}{b-a} & a \leq x \leq b \\
1 & b < x < c \\
\frac{d-x}{d-c} & c \leq x \leq d \\
0 & x > d 
\end{cases}
\]

\[
\mu_A(x) = \begin{cases} 
0 & x < a \\
\frac{1}{2} \left( 1 - \cos \left( \frac{\pi x-a}{b-a} \right) \right) & a \leq x \leq b \\
1 & b < x < c \\
\frac{1}{2} \left( 1 + \cos \left( \frac{\pi x-c}{d-c} \right) \right) & c \leq x \leq d \\
0 & x > d 
\end{cases}
\]
Trapezoidal Membership Functions

LeftTrapezoid
Left_Slope = 0
Right_Slope = \frac{1}{A - B}

CASE 1: \( X < a \)
Membership Value = 1

CASE 2: \( X \geq b \)
Membership Value = 0

CASE 3: \( a < x < b \)
Membership Value = Right_Slope \times (X - b)
RightTrapezoid
Left_Slope = 1 / (B - A)
Right_Slope = 0
CASE 1: X <= a
Membership Value = 0
CASE 2: X >= b
Membership Value = 1
CASE 3: a < x < b
Membership Value = Left_Slope * (X - a)
Regular Trapezoid
Left_Slope = 1 / (B - A)
Right_Slope = 1 / (C - D)
CASE 1: X <= a Or X >= d
Membership Value = 0
CASE 2: X >= b And X <= c
Membership Value = 1
CASE 3: X >= a And X <= b
Membership Value = Left_Slope * (X - a)
CASE 4: (X >= c) And (X <= d)
Membership Value = Right_Slope * (X - d)
The S-function can be used to define fuzzy sets
• \( S(x, \ a, \ b, \ c) = \)
• 0 for \( x \leq a \)
• \( 2(x-a/c-a)^2 \) for \( a \leq x \leq b \)
• \( 1 -2(x-c/c-a)^2 \) for \( b \leq x \leq c \)
• 1 for \( x \geq c \)
Membership functions: \( \Pi \)-Function

\[ \Pi(x, a, b) = \]

- \( S(x, b-a, b-a/2, b) \) for \( x \leq b \)
- \( 1 - S(x, b, b+a/2, a+b) \) for \( x \geq b \)
Membership functions: S-function

- The S-function can be used to define fuzzy sets

\[ S(x, a, b, c) = \]
- 0 for \( x \leq a \)
- \( 2(x-a/c-a)^2 \) for \( a \leq x \leq b \)
- \( 1 - 2(x-c/c-a)^2 \) for \( b \leq x \leq c \)
- 1 for \( x \geq c \)
Membership functions: $\Pi$-Function

$$\Pi(x, a, b) =$$

- $S(x, b-a, b-a/2, b)$ for $x \leq b$
- $1 - S(x, b, b+a/2, a+b)$ for $x \geq b$

E.g., close (to $a$)
• Simple membership functions
• Piecewise linear: triangular etc.
• Easier to represent as easy to represent and calculate
  \[ \Rightarrow \text{saves computation} \]
Linguistic Hedges

- Modifying the meaning of a fuzzy set using hedges such as very, more or less, slightly, etc.

- "Very $F" = F^2$
- "More or less $F" = F^{1/2}$. 
- etc.
## Representation of hedges in fuzzy logic

<table>
<thead>
<tr>
<th>Hedge</th>
<th>Mathematical Expression</th>
<th>Graphical Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A little</td>
<td>$[\mu_A(x)]^{1.3}$</td>
<td>![A little Triangle]</td>
</tr>
<tr>
<td>Slightly</td>
<td>$[\mu_A(x)]^{1.7}$</td>
<td>![Slightly Triangle]</td>
</tr>
<tr>
<td>Very</td>
<td>$[\mu_A(x)]^2$</td>
<td>![Very Triangle]</td>
</tr>
<tr>
<td>Extremely</td>
<td>$[\mu_A(x)]^3$</td>
<td>![Extremely Triangle]</td>
</tr>
</tbody>
</table>
## Representation of hedges in fuzzy logic (continued)

<table>
<thead>
<tr>
<th>Hedge</th>
<th>Mathematical Expression</th>
<th>Graphical Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very very</td>
<td>$[\mu_A(x)]^4$</td>
<td><img src="image1" alt="Graph" /></td>
</tr>
<tr>
<td>More or less</td>
<td>$\sqrt{\mu_A(x)}$</td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td>Somewhat</td>
<td>$\sqrt{\mu_A(x)}$</td>
<td><img src="image3" alt="Graph" /></td>
</tr>
<tr>
<td>Indeed</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2 [\mu_A(x)]^2$ if $0 \leq \mu_A \leq 0.5$</td>
<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td></td>
<td>$1 - 2 [1 - \mu_A(x)]^2$ if $0.5 &lt; \mu_A \leq 1$</td>
<td><img src="image5" alt="Graph" /></td>
</tr>
</tbody>
</table>
Complement

__Crisp Sets:__ Who does not belong to the set?
__Fuzzy Sets:__ How much do elements not belong to the set?

The complement of a set is an opposite of this set. For example, if we have the set of *tall men*, its complement is the set of *NOT tall men*.

When we remove the tall men set from the universe of discourse, we obtain the complement.

If $A$ is the fuzzy set, its complement $\neg A$ can be found as follows:

$$\mu_{\neg A}(x) = 1 - \mu_A(x)$$
Crisp Sets: Which element belongs to both sets?
Fuzzy Sets: How much of the element is in both sets?

In classical set theory, an intersection between two sets contains the elements shared by these sets. For example, the intersection of the set of tall men and the set of fat men is the area where these sets overlap.

In fuzzy sets, an element may partly belong to both sets with different memberships. A fuzzy intersection is the lower membership in both sets of each element. The fuzzy intersection of two fuzzy sets $A$ and $B$ on universe of discourse $X$:

$$\mu_{A \cap B}(x) = \min [\mu_A(x), \mu_B(x)] = \mu_A(x) \cap \mu_B(x),$$

where $x \in X$
The union of two crisp sets consists of every element that falls into either set. For example, the union of *tall men* and *fat men* contains all men who are tall OR fat.

In fuzzy sets, the union is the reverse of the intersection. That is, the union is the largest membership value of the element in either set.

The fuzzy operation for forming the union of two fuzzy sets $A$ and $B$ on universe $X$ can be given as:

$$\mu_{A \cup B}(x) = \max [\mu_A(x), \mu_B(x)] = \mu_A(x) \cup \mu_B(x),$$

where $x \in X$
Operations of fuzzy sets

![Operations of fuzzy sets](image)

- **Complement**: The complement of a fuzzy set $A$ is denoted by $\overline{A}$. It represents the set of all elements $x$ for which the membership function $\mu_x$ is less than 1.

- **Containment**: Two fuzzy sets $A$ and $B$ are said to be contained within each other if $\mu_A(x) \leq \mu_B(x)$ for all $x$.

- **Intersection**: The intersection of two fuzzy sets $A$ and $B$, denoted by $A \cap B$, is the set of all elements $x$ for which $\mu_A(x) = \mu_B(x)$.

- **Union**: The union of two fuzzy sets $A$ and $B$, denoted by $A \cup B$, is the set of all elements $x$ for which $\mu_A(x) + \mu_B(x) - \mu_A(x) \cap \mu_B(x)$.
Step 1. Evaluate the antecedent for each rule

1. Fuzzify inputs

2. Apply OR operator (máx)

Antecedent of the rule

service is excellent or food is delicious

Input 1

service = 3

Input 2

food = 8

Result

0.7
Step 2. Obtain each rule's conclusion

1. Fuzzify inputs

2. Apply OR operator (max)

3. Apply implication operator (min.)

If service is excellent or food is delicious then tip is generous

Result of implication
Step 3. Aggregate conclusions
Step 4. Defuzzification

5. Defuzzify the aggregate output (centroid)

\[ g = \frac{\sum_{i=1}^{9} x_i \cdot u(x_i)}{\sum_{i=1}^{9} u(x_i)} = 16.7 \]

Result of defuzzification

\[ \text{tip} = 16.7\% \]
1. Fuzzify inputs

- service is poor
- food is rancid

then tip is cheap

2. Apply OR operator (max)

rule 2 has no dependency on input 2

if service is good

then tip is average

3. Apply implication operator (min.)

if service is excellent

or food is delicious

then tip is generous

4. Apply aggregation method (max)

5. Defuzzify (centroid)
Algorithm 1 Fuzzy logic algorithm

1. Define the linguistic variables and terms (initialization)
2. Construct the membership functions (initialization)
3. Construct the rule base (initialization)
4. Convert crisp input data to fuzzy values using the membership functions (fuzzification)
5. Evaluate the rules in the rule base (inference)
6. Combine the results of each rule (inference)
7. Convert the output data to non-fuzzy values (defuzzification)
Defuzzifier

- Converts the fuzzy output of the inference engine to crisp using membership functions analogous to the ones used by the fuzzifier.
- Five commonly used defuzzifying methods:
  - Centroid of area (COA)
  - Bisector of area (BOA)
  - Mean of maximum (MOM)
  - Smallest of maximum (SOM)
  - Largest of maximum (LOM)
Average maximum
First maximum

![Diagram showing maximum points labeled A1, A2, A3, A4, A5.](image-url)
Last maximum
Center of gravity

\[ u_0 = \frac{\int \mu(u) u \, du}{\int \mu(u) \, du} \]
Fuzzy inference rules (mamdami)

The example of the insurance company

1. if driver ∈ young and car power ∈ high then risk ∈ high
2. if driver ∈ young and car power ∈ average then risk ∈ high
3. if driver ∈ medium and car power ∈ high then risk ∈ average
4. if driver ∈ medium and car power ∈ average then risk ∈ low

Driver = \{young, medium, old\}
Car power = \{small, average, high\}
risk = \{low, average, high\}
Age of the driver

\[ \mu_{\text{young}}(x) = \begin{cases} 
1 & x \leq 30 \\
0 & x \geq 40 \\
\frac{x-40}{30-40} & 30 < x < 40 
\end{cases} \]

\[ \mu_{\text{medium}}(x) = \begin{cases} 
0 & x \leq 30 \text{ lub } x \geq 50 \\
\frac{x-30}{40-30} & 30 < x < 40 \\
\frac{50-x}{50-40} & 40 < x < 50 
\end{cases} \]

\[ \mu_{\text{old}}(x) = \begin{cases} 
0 & x \leq 40 \\
1 & x \geq 50 \\
\frac{x-40}{50-40} & 40 < x < 50 
\end{cases} \]
Car power

\[ \mu_{\text{small}}(x) = \begin{cases} 
1 & x \leq 70 \\
0 & x \geq 120 \\
\frac{x - 120}{70 - 120} & 70 < x < 120 
\end{cases} \]

\[ \mu_{\text{average}}(x) = \begin{cases} 
0 & x \leq 70 \text{ lub } x \geq 170 \\
\frac{x - 70}{120 - 70} & 70 < x < 120 \\
\frac{170 - x}{170 - 120} & 120 < x < 170 
\end{cases} \]

\[ \mu_{\text{high}}(x) = \begin{cases} 
0 & x \leq 120 \\
1 & x \geq 170 \\
\frac{x - 120}{170 - 120} & 120 < x < 170 
\end{cases} \]
Insurance company risk

<table>
<thead>
<tr>
<th>risk</th>
<th>low</th>
<th>average</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>30</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>35</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>40</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Assurance risk

\[
\mu_{low}(x) = \begin{cases} 
1 & x \leq 10 \\
0 & x \geq 20 \\
\frac{x-10}{20-10} & 10 < x < 20 
\end{cases}
\]

\[
\mu_{avg}(x) = \begin{cases} 
0 & x \leq 10 lub x \geq 30 \\
\frac{x-10}{20-10} & 10 < x < 20 \\
\frac{30-x}{30-20} & 20 < x < 30 
\end{cases}
\]

\[
\mu_{high}(x) = \begin{cases} 
0 & x \leq 20 \\
1 & x \geq 30 \\
\frac{x-20}{30-20} & 20 < x < 30 
\end{cases}
\]
Fuzzy inference rules (mamdami)

The example of the insurance company

- R1: if driver ∈ young and car power ∈ high then risk ∈ high
- R2: if driver ∈ young and car power ∈ average then risk ∈ high
- R3: if driver ∈ medium and car power ∈ high then risk ∈ average
- R4: if driver ∈ medium and car power ∈ average then risk ∈ low

Driver age = 38
Car power = 166 KM

1st with degree min(0.2, 0.92) = 0.2
2nd with degree min(0.2, 0.08) = 0.08
3rd with degree min(0.8, 0.92) = 0.8
4th with degree min(0.8, 0.08) = 0.08
We find rules to activate

<table>
<thead>
<tr>
<th>AGE</th>
<th>young</th>
<th>medium</th>
<th>old</th>
<th>CarPower small</th>
<th>average</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>30</td>
<td>0</td>
</tr>
<tr>
<td>35</td>
<td>0,5</td>
<td>0,5</td>
<td>0</td>
<td>0</td>
<td>80</td>
<td>0,8</td>
</tr>
<tr>
<td>40</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>130</td>
<td>0,8</td>
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<tr>
<td>55</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>180</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>65</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>230</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>38</td>
<td>0,2</td>
<td>0,8</td>
<td>0</td>
<td>166</td>
<td>0</td>
<td>0,08</td>
</tr>
</tbody>
</table>

1. if driver ∈ young and car power ∈ high then risk ∈ high
2. if driver ∈ young and car power ∈ average then risk ∈ high
3. if driver ∈ medium and car power ∈ high then risk ∈ average
4. if driver ∈ medium and car power ∈ average then risk ∈ low

We choose the MIN value in each rule...
We find rules to activate

1. if driver ∈ young and car power ∈ high then risk ∈ high
2. if driver ∈ young and car power ∈ average then risk ∈ high
3. if driver ∈ medium and car power ∈ high then risk ∈ average
4. if driver ∈ medium and car power ∈ average then risk ∈ low

2 rules with the same decision but different values....
We find rules to activate

1. if driver ∈ young and car power ∈ high then risk ∈ high
2. if driver ∈ young and car power ∈ average then risk ∈ high
3. if driver ∈ medium and car power ∈ high then risk ∈ average
4. if driver ∈ medium and car power ∈ average then risk ∈ low

2 rules with the same decision but different values....
So we have to choose one decision by calculation MAX value....
We find rules to activate

\[
\mu_{\text{risk}=\text{high}}(age = 38, \text{car power}=166) = \min \left( \mu_{\text{young}}(38), \mu_{\text{high}}(166) \right) = \min(0.2, 0.92) = 0.2
\]

\[
\mu_{\text{risk}=\text{high}}(age = 38, \text{car power}=166) = \min \left( \mu_{\text{young}}(38), \mu_{\text{average}}(166) \right) = \min(0.2, 0.08) = 0.08
\]

\[
\mu_{\text{risk}=\text{avg}}(age = 38, \text{car power}=166) = \min \left( \mu_{\text{medium}}(38), \mu_{\text{high}}(166) \right) = \min(0.8, 0.92) = 0.8
\]

\[
\mu_{\text{risk}=\text{low}}(age = 38, \text{car power}=166) = \min \left( \mu_{\text{medium}}(38), \mu_{\text{average}}(166) \right) = \min(0.8, 0.08) = 0.08
\]
We find rules to activate

\[ \mu_{risk=\text{high}}(age = 38, \text{car power} = 166) = \min \left( \mu_{\text{young}}(38), \mu_{\text{high}}(166) \right) = \min(0.2, 0.92) = 0.2 \]

\[ \mu_{risk=\text{high}}(age = 38, \text{car power} = 166) = \min \left( \mu_{\text{young}}(38), \mu_{\text{average}}(166) \right) = \min(0.2, 0.08) = 0.08 \]

\[ \mu_{risk=\text{avg}}(age = 38, \text{car power} = 166) = \min \left( \mu_{\text{medium}}(38), \mu_{\text{high}}(166) \right) = \min(0.8, 0.92) = 0.8 \]

\[ \mu_{risk=\text{low}}(age = 38, \text{car power} = 166) = \min \left( \mu_{\text{medium}}(38), \mu_{\text{average}}(166) \right) = \min(0.8, 0.08) = 0.08 \]
defuzzification

- Center of gravity
- Mean of maximum
- First maximum
- Last maximum
Fuzzy inference rules (mamdami)

Risk is

- High with degree $= \max(0.2, 0.08) = 0.2$
- Average with degree $= 0.8$
- Low with degree $= 0.08$

Risk value as a center of gravity:

$$r = \frac{\int x \cdot \mu_r(x) \, dx}{\int \mu_r(x) \, dx}$$
Fuzzy inference rules (mamdami)

Risk is: High with degree $\max(0.2,0.08) = 0.2$
Average with degree $= 0.8$
Low with degree $0.08$

Risk value as a center of gravity:

$$cog = \frac{10 \times 0.2 + 20 \times 0.8 + 30 \times 0.08}{0.2 + 0.8 + 0.08} = 18.89$$
Mean maximum

- 20
First maximum

- 17.5
Last maximum

- 22.5
CONCLUSION

• Fuzzy logic provides an alternative way to represent linguistic and subjective attributes of the real world in computing.

• It is able to be applied to control systems and other applications in order to improve the efficiency and simplicity of the design process.
Homework

1. if driver ∈ young and car power ∈ high then risk ∈ high
2. if driver ∈ young and car power ∈ average then risk ∈ high
3. if driver ∈ medium and car power ∈ high then risk ∈ average
4. if driver ∈ medium and car power ∈ average then risk ∈ low

Age of the driver = 35, Car power = 150. What is the assurance risk?