Fuzzy logic - exercices
Book online

http://profs.basu.ac.ir/rashidykanan/free_space/a%20course%20in%20fuzzy%20systems%20and%20control_part%201.pdf
Intersection

Given two fuzzy sets $A$ and $B$, the membership function of the intersection $C = A \cap B$ is defined as

$$m_C(x) = \min \{m_A(x), m_B(x)\}, \ x \in X \tag{4}$$

Union

Given two fuzzy sets $A$ and $B$, the membership function of the union $D = A \cup B$ is defined as

$$m_D(x) = \max \{m_A(x), m_B(x)\}, \ x \in X \tag{5}$$

Complement

And finally, the membership function of the complement $A^c$ of a fuzzy set $A$ is defined as

$$m_{A^c}(x) = 1 - m_A(x), \ x \in X \tag{6}$$

The intuition behind these definitions is fairly obvious. In the case where the membership functions are characteristic functions having values of 0 and 1, we have the definitions of classical set theory.

To illustrate these definitions let us look at an example. In addition to the set of “comfortable house for a 4-person family” we define a “large house”, $B$ as follows:

$$B = \{(T3, 0.2), (T4, 0.4), (T5, 0.6), (T6, 0.8), (T7, 1), (T8, 1)\} \tag{7}$$

Thus we get for intersection, union, and complement:

$$A \cap B = \{(T3, 0.2), (T4, 0.4), (T5, 0.6), (T6, 0.3)\}$$

$$A \cup B = \{(T1, 0.2), (T2, 0.5), (T3, 0.8), (T4, 1), (T5, 0.7), (T6, 0.8), (T7, 1), (T8, 1)\}$$

$$B^c = \{(T1, 1), (T2, 1), (T3, 0.8), (T4, 0.6), (T5, 0.4), (T6, 0.2), (T9, 1), (T10, 1)\}$$
Exercise nr 1

a. You are assigned the task of identifying images in an overhead reconnaissance photograph. The two fuzzy sets representing a car image and a truck image are defined as:

\[ \text{Car} = \{0.5 / \text{truck}, 0.4 / \text{motor}, 0.3 / \text{boat}, 0.9 / \text{car}, 0.1 / \text{house}\} \]
\[ \text{Truck} = \{1 / \text{truck}, 0.1 / \text{motor}, 0.4 / \text{boat}, 0.4 / \text{car}, 0.2 / \text{house}\} \]

Find the following:

1. Car \cup \text{Truck}
2. Car \cap \text{Truck}
3. \text{not}(\text{Car})
4. \text{Car} \cap \text{not(Truck)}
5. \text{Car} \cup \text{not(\text{Car})}
6. \text{Car} \cap \text{not(\text{Car})}
d. The following fuzzy function was used to calculate membership values for the set healthy. A membership value of 1 is healthy; a membership value of 0 is not healthy; a membership value between 0 and 1 is the degree of membership in the healthy set.

\[
\text{healthy}(x) = \begin{cases} 
0, & \text{if } \text{bmi}(x) < 18 \\
(bmi(x)-18)/2, & \text{if } 18 \leq \text{bmi}(x) \leq 20 \\
1, & \text{if } 20 < \text{bmi}(x) < 25 \\
(27-bmi(x))/2, & \text{if } 25 \leq \text{bmi}(x) \leq 27 \\
0, & \text{if } \text{bmi}(x) > 27 
\end{cases}
\]

BMI values that range from 20 to 25 are members of the healthy set (1). BMI values greater than 27 or less then 18 are not members of the healthy set (0). BMI values close to the healthy range (20 to 25) are a value between 0 and 1. For example, a BMI of 19.6 is 0.8 degree of membership in the healthy set.

1. Draw the graphic for the healthy set, representing the values, healthy and unhealthy.
2. What is the degree of membership to the fuzzy set healthy of John who has a BMI of 26.2? And to the fuzzy set unhealthy?
3. Calculate now your BMI (weight in kg/(length in m)^2). What is your degree of membership for the healthy set?
Exercise nr 3

Consider the fuzzy subset of person heights considered as *tall* person heights. Assume that it is characterized by the membership function depicted below.

Define the corresponding membership function: 

\[ \mu_{\text{Tall}}(x) = \begin{cases} 
0 & \text{if } x < 160 \\
\frac{x - 160}{50} & \text{if } 160 \leq x < 210 \\
1 & \text{if } x \geq 210 
\end{cases} \]

Assume, Peter’s height is 180 cm. To which degree is Peter tall?
Exercise nr 4

Exercise 2. Mamdani rule

Hand-in: everything.

Consider the following fuzzy expert system:

RULE1: IF temperature is hot or warm, THEN the swimming pool is crowded.
RULE2: IF temperature is cold, THEN the swimming pool is quiet.

The membership function in the universes of discourse are given as

\[
\mu_{\text{Hot}}(x) = \begin{cases} 
0, & \text{if } x < 15 \\
\frac{x-15}{10}, & \text{if } 15 \leq x \leq 25 \\
1, & \text{if } x > 25 
\end{cases}
\]

\[
\mu_{\text{Warm}}(x) = \begin{cases} 
-(\frac{x-25}{10}), & \text{if } 15 \leq x \leq 25 \\
\frac{x-7}{8}, & \text{if } 7 \leq x < 15 \\
0, & \text{if } x < 7 \text{ or } x > 25 
\end{cases}
\]

\[
\mu_{\text{Cold}}(x) = \begin{cases} 
1, & \text{if } x < 7 \\
1 - \frac{x-7}{8}, & \text{if } 7 \leq x \leq 15 \\
0, & \text{if } x > 15 
\end{cases}
\]

\[
\mu_{\text{Crowd}}(x) = \begin{cases} 
0, & \text{if } x < 100 \\
\frac{x-100}{400}, & \text{if } 100 \leq x \leq 500 \\
1, & \text{if } 500 \leq x \leq 800 
\end{cases}
\]

\[
\mu_{\text{Quiet}}(x) = \begin{cases} 
1, & \text{if } x < 100 \\
-(\frac{x-500}{400}), & \text{if } 100 \leq x \leq 500 \\
0, & \text{if } 500 \leq x \leq 800 
\end{cases}
\]

a. Which are the linguistic variables and the linguistic values?
b. Draw the graphics for the membership functions of temperature and number of customers in the swimming pool.
c. Suppose the temperature is 21 degrees. Apply the Mamdani implication rule to determine the expected number of clients in the swimming pool. Describe carefully each of the steps.
Exercise nr 5

Draw Some Representative Fuzzy Membership Functions

The Triangular membership function is defined as

\[ \mu_A(x) = \text{triangular}(x; a, b, c) = \begin{cases} 
0, & x \leq a \\
\frac{x-a}{b-a}, & a \leq x \leq b \\
\frac{c-x}{c-b}, & b \leq x \leq c \\
0, & c \leq x 
\end{cases} \]

The Trapezoid membership function is defined as

\[ \mu_A(x) = \text{trapezoid}(x; a, b, c, d) = \begin{cases} 
0, & x \leq a \\
\frac{x-a}{b-a}, & a \leq x \leq b \\
1, & b \leq x \leq c \\
\frac{d-x}{d-c}, & c \leq x \leq d \\
0, & d \leq x 
\end{cases} \]
Questions

a. What is the normal range of truth values in Boolean logic? And in fuzzy logic?
b. Would it be correct to call fuzzy logic multi-valued?
c. Define a linguistic variable and its values. Give an example and draw the fuzzy set for that variable. How can a hedge modify this variable, and its fuzzy set?
quizz

http://www.doc.ic.ac.uk/~nd/surprise_96/journal/vol3/sbaa/test.html